1 Introduction

Answers to Questions and Problems

1. If an arbitrage opportunity did exist in a market, how would traders react? Would the arbitrage opportunity persist? If not, what factors would cause the arbitrage opportunity to disappear?

   Traders are motivated by profit opportunities, and an arbitrage opportunity represents the chance for riskless profit without investment. Therefore, traders would react to an arbitrage opportunity by trading to exploit the opportunity. They would buy the relatively underpriced asset and sell the relatively overpriced asset. The arbitrage opportunity would disappear, because the presence of the arbitrage opportunity would create excess demand for the underpriced asset and excess supply of the overpriced asset. The arbitrageurs would continue their trading until the arbitrage opportunity disappeared.

2. Explain why it is reasonable to think that prices in a financial market will generally be free of arbitrage opportunities.

   Generally arbitrage opportunities will not be available in financial markets because well-informed and intelligent traders are constantly on the lookout for such chances. As soon as an arbitrage opportunity appears, traders trade to take advantage of the opportunity, causing the mispricing to be corrected.

3. Explain the difference between a derivative instrument and a financial derivative.

   A derivative is a financial instrument or security whose payoffs depend on any underlying asset. A financial derivative is a financial instrument or security whose payoffs depend on an underlying financial instrument or security.

4. What is the essential feature of a forward contract that makes a futures contract a type of forward contract?

   A forward contract always involves the contracting at one moment in time with the performance under the contract taking place at a later date. Thus, futures represent a kind of forward contract under this definition.

5. Explain why the purchaser of an option has rights and the seller of an option has obligations.

   The purchaser of an option makes a payment that is the consideration given to acquire certain rights. By contrast, the seller of an option receives payment at the time of sale and undertakes certain obligations in return for that payment.

6. In a futures contract, explain the rights and obligations of the buyer or seller. How does this compare with an option contract?

   In a futures contract, both the buyer and the seller have both obligations and rights. The buyer of a futures contract promises to make payment and take delivery at a future date, while the seller of a futures contract

promises to make delivery and receive payment at a future date. This contrasts with the option market in which the buyer has only rights and the seller has only obligations following the original transaction.

7. Explain the difference between an option on a physical good and an option on a futures.

   An option on a physical good gives the owner the right to buy the physical good by paying the exercise price, and the seller of an option is obligated to deliver the good. When the owner of either a call or put futures option exercises, no delivery of any physical good occurs. Instead, both the buyer and seller of the futures option receive a position in a futures contract.

8. What is the essential feature of a swap agreement?

   Essentially, a swap agreement obligates the two counterparties to make payments to each other over time. These payment streams can be tied to the value of interest rate sensitive instruments, to foreign currency values, to the fluctuating value of physical commodities, or to any other item of value.

9. Distinguish between interest rate swaps and currency swaps.

   In an interest rate swap, one party pays to another a fixed rate of interest, while the second party pays a floating rate of interest. Generally, no principal changes hands, but the promised payments are tied to some measure of interest rates. In a currency swap, payments between the two counterparties are typically made in different currencies, and there is often an initial exchange of principal amounts in different currencies at the outset of the swap agreement.

10. What is a complete market? Can you give an example of a truly complete market? Explain.

    A **complete market** is a market in which any and all identifiable payoffs can be obtained by trading the securities available in the market. A complete market is essentially a theoretical ideal and is unlikely to be observed in practice.

11. Explain how the existence of financial derivatives enhances speculative opportunities for traders in our financial system.

    Speculators can use financial derivatives to profit from their correct anticipation of changes in interest rates, currency values, stock market levels, and so on. Financial derivatives are particularly powerful speculative instruments because they can be managed to give specific risk exposures while avoiding risks that are unwanted. In addition, financial derivatives markets are often more liquid than the underlying markets, and financial derivatives can be traded with lower transaction costs in many instances.

12. If financial derivatives are as risky as their reputation indicates, explain in general terms how they might be used to reduce a preexisting risk position for a firm.

    While an outright position in a financial derivative considered in isolation generally embodies considerable risk, these instruments can be used to offset other preexisting risks that a firm might face. For example, a savings and loan association might face potential losses due to rising interest rates, and this risk might arise from the normal conduct of its business. Such an association could use interest rate futures, options on interest rate futures, or swap agreements to offset that preexisting risk. Properly managed, financial derivatives can reduce a preexisting business risk through hedging.

13. Consider the following three securities. Let us assume that at one period in the future the market will move either up or down. This movement in the market produces the following payoffs for the three securities. (Note: This problem is a challenge problem and presumes some familiarity with arbitrage concepts. The issues raised by this problem are explored directly in Chapter 13.)

Security	Current Price	Payoff When the Market Moves Down	Payoff When the Market Moves Up
A	\$35	\$25	\$50
B	\$30	\$15	\$60
C	\$40	\$19	\$56

A. Construct a portfolio consisting of securities A and B that replicates the payoffs on security C in both the up and down states subject to the constraint that the sum of the commitments to the two securities (A and B) is one. In other words, construct a synthetic share of security C. Assume that there are no restrictions associated with short selling any of the securities.

To create a synthetic share of security C, we must construct a portfolio of securities A and B that has the same payoffs as security C in both the up and down states of the market. That is, we must construct a portfolio with the following characteristics:

	Synthetic Security (Portfolio of Securities A and B)	Security C
Payoff when the market moves down	$\$25 \times W_A + \$15 \times W_B$	\$19
Payoff when the market moves up	$\$50 \times W_A + \$60 \times W_B$	\$56

subject to the constraint that $W_A + W_B = 1$. W_A and W_B are the proportions of the portfolio's value committed to instruments A and B. The two equations in the table constitute two equations in two unknowns. We solve them simultaneously, by multiplying the first equation times -2.0 and summing them to solve for W_B:

$$-2.0 \times (\$25 \times W_A + \$15 \times W_B) + \$50 \times W_A + \$60 \times W_B = -2.0 \times \$19 + \$56$$
$$W_A = \$18/\$30 = 0.6$$

Therefore, $W_B = 0.4$, because $W_A + W_B = 1$.

B. What are the commitments to securities A and B?

Forty percent of the investor's wealth is committed to security A, and 60 percent of the investor's wealth is committed to security B, $W_A = .4$ and $W_B = .6$.

C. How much does it cost to construct a synthetic share of security C? Compare this cost with the market price of security C. Which security is cheaper?

The cost of constructing the synthetic security is:

$$.4 \times \$35 + .6 \times \$30 = \$32$$

Security C is trading at a price of \$40. The synthetic security is cheaper.

D. Explain the transactions necessary to engage in riskless arbitrage. Explain why these transactions constitute a riskless arbitrage opportunity. How much profit can an investor make in this riskless arbitrage?

These prices present an opportunity for the investor to engage in riskless arbitrage. The synthetic security constructed to produce the same payoffs as security C in all states of nature is cheaper than security C itself. To take advantage of this situation, we must simultaneously sell the overpriced, expensive security and buy the underpriced, cheap security. That is, we must simultaneously sell security C and buy the synthetic security. Arbitrage is transacting to secure a riskless profit without investment. For this to be a riskless arbitrage opportunity, we must simultaneously enter into the purchase and sale transactions, and the transactions

must be of the same scale. That is, we must buy and sell the same number of units of the securities. Note that the math of the problem requires one to sell fractions of a unit of a security. However, institutional constraints do not permit the sale or purchase of fractional units of a security. We can easily solve this problem by scaling up our purchase and sale transactions by 10. That is, we will sell 10 units of security C and buy 10 units of the synthetic security. Since we are assuming that the investor does not own security C, the investor will be engaging in short selling.

Riskless arbitrage transactions:

	Transactions	Cash Flows
Security C	Sell 10 units of C at $40.	+$400
Synthetic security	Buy 4 units of A at $35.	−$140
	Buy 6 units of B at $30.	−$180
Profit		+$80

The profit on the transactions is $80 on 10 units of the securities, or $8 profit per unit. The profit could be increased to infinity by scaling up the size of all of these transactions.

E. Explain why we do not have to worry about future obligations in a properly constructed riskless arbitrage transaction.

Consider the following:

	Obligation Associated with the Arbitrage Transactions	Assets Held as a Result of the Arbitrage Transactions	Cash Flows
Payoff when the market moves down	Short selling 10 units of security C obligates the trader to deliver 10 units of security C worth $19 per unit.	The investor holds 10 units of the synthetic security worth $19 per unit.	−$190 + $190 = $0
Payoff when the market moves up	Short selling 10 units of security C obligates the trader to deliver 10 units of security C worth $56 per unit.	The investor holds 10 units of the synthetic security worth $56 per unit.	−$560 + $560 = $0

The arbitrage transactions were structured such that the value of the investor's assets are equal to the value of the investor's obligations. In addition, the synthetic security was constructed such that the payoffs on the synthetic security were equal to the payoffs from investing in security C in both states of nature.

F. Explain why we would not expect such a structure of prices to exist in the marketplace.

If these prices were observed in the market, the investor could earn a certain profit with no commitment of her own resources. Persistence of such mispricing would permit all investors to become infinitely wealthy through riskless arbitrage.

G. Explain the purpose of short selling in riskless arbitrage. Discuss the impact on the investor's ability to engage in riskless arbitrage of regulations that limit an investor's access to the proceeds from a short sale transaction.

The purpose of short selling in arbitrage transactions is to finance the purchase of the "cheap" asset through the sale of the "expensive" asset. In arbitrage we sell high to finance the transaction. Any constraints that limit the investor's access to funds from short selling limit the investor's ability to engage in riskless arbitrage. For example, if margin requirements are such that the investor does not have access to any of the proceeds from short selling, then the investor will have to use her personal wealth to pay for the securities she

has purchased "cheap." In this environment, the investor is no longer engaging in riskless arbitrage, because investment is now required and the arbitrage transactions are no long self-financing.

H. Construct a portfolio consisting of securities B and C that replicates the payoffs on security A in both the up and down states subject to the constraint that the sum of the commitments to the two securities (B and C) is one. In other words, construct a synthetic share of security A. Assume that there are no restrictions associated with short selling any of the securities.

To create a synthetic share of security A, we must construct a portfolio of securities B and C that has the same payoffs as security A in both the up and down states of the market. That is, we must construct a portfolio with the following characteristics:

	Synthetic Security (Portfolio of Securities B and C)	Security A
Payoff when the market moves down	$\$15 \times W_B + \$19 \times W_C$	\$25
Payoff when the market moves up	$\$60 \times W_B + \$56 \times W_C$	\$50

subject to the constraint that $W_B + W_C = 1$.

Multiplying the first equation in the table by -4 and adding the resulting equation to the second equation, we can solve the two equations in the table simultaneously for W_C.

$$-4 \times (\$15 \times W_B + \$19 \times W_C) + \$60 \times W_B + \$56 \times W_C = -4 \times \$25 + \$50$$

$W_C = 2.5$, and $W_B = -1.5$, where $W_B + W_C = 1$. That is, to construct synthetic security A, we must short sell security B and invest the proceeds from the short sale of security B in security C.

I. What are the commitments to securities B and C?

Construction of the synthetic security required the short sale of security B to finance the purchase of additional units of security C. Thus, the investor commits more than 100 percent of his wealth to security C. Specifically 250 percent of the investor's wealth is committed to security C financed by short sales of security B in an amount equal to 150 percent of the investor's wealth, $W_B = -1.5$ and $W_C = 2.5$.

J. How much does it cost to construct a synthetic share of security A? Compare this cost with the market price of security A. Which security is cheaper?

The cost of constructing the synthetic security is \$55, $-1.5 \times \$30 + 2.5 \times \$40 = \$55$. Security A is trading at a price of \$35. Security A is cheaper than the synthetic security.

K. Explain the transactions necessary to engage in a riskless arbitrage. How much profit can an investor make in this riskless arbitrage? Assume that one trades 10 shares of security A in constructing the arbitrage transactions.

These prices present an opportunity for the investor to engage in riskless arbitrage. The synthetic security constructed to produce the same payoffs as security A in all states of nature is more expensive than security A itself. Therefore, we must sell the overpriced synthetic security and buy security A itself.

Riskless arbitrage transactions:

	Transactions	Cash Flows
Sell synthetic security	Buy 15 units of B at \$30.	−\$450
	Sell 25 units of C at \$40.	+\$1,000
Buy security A	Buy 10 units of A at \$35.	−\$350
Profit		+\$200

Short selling the synthetic security requires the investor to purchase security B, and sell security C. The profit on the transactions is $200 on 10 units of security A, or $20 profit per unit.

L. Discuss the differences between the transactions necessary to capture the arbitrage profit when creating a synthetic share of security A and the arbitrage transactions undertaken to capture the arbitrage profit when creating a synthetic share of security C.

In the first problem, the construction of the synthetic security required positive commitments by the investor to each security, that is, the portfolio weights were positive. In addition, to capture the profit available from the mispricing of the securities, we sold the actual security short and bought the synthetic security. In the second problem, construction of the synthetic security required the investor to sell security B short to finance purchases of security C. We then short sold the synthetic security and bought the actual security to reap the arbitrage profits.